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SOUND PROPAGATION IN POLYDISPERSED GAS SUSPENSIONS

N. A. Gumerov and A. I. Ivandaev

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The majority of studies of acoustics of gas suspensions have investigated propagation of linear and slightly nonlinear waves in monodispersed mixtures [1-5]. The effect of polydispersion on propagation of linear monochromatic waves was first studied in [6]. However only the simple case of low mass content of the suspended phase was considered, in which case the contribution of particles of a given size to sound dispersion and dissipation is actually proportional to their mass fraction in the mixture. The present study will investigate unique features of sound wave propagation in polydispersed gas and vapor suspensions for arbitrary (not necessarily small) mass content of suspended particles or droplets for the first time. Some of the results were reported previously in [7].

1. General Considerations. Real gas suspensions of both natural and artificial origin are usually not monodispersed. They contain particles of quite differing sizes, which often differ greatly from each other. The dispersion composition of such mixtures can be characterized at each point in space by a particle distribution function over size $N(a, \mathbf{r}, t)$, as well as the minimum $a_{\min}(\mathbf{r}, t)$ and maximum $a_{\max}(\mathbf{r}, t)$ radii. We have

$$dn(a, \mathbf{r}, t) = N(a, \mathbf{r}, t)\,da, \quad n(\mathbf{r}, t) = \int\limits_{a_{\min}}^{a_{\max}} N(a, \mathbf{r}, t)\,da.$$

Here a is the particle radius, $\mathbf{r}$ is the radius vector of the point, t is time, dn is the number of particles per unit volume having radii from a to a + da, n is the total number of particles of all sizes per unit volume of mixture at the space-time point $(\mathbf{r}, t)$.

We will consider the quite general case of a mixture with phase transitions at phase separation boundaries. In the process of motion of such a mixture, the particle (droplet) dis-

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tribution over size changes not only because of motion of droplets of various sizes relative to each other, but also because of direct change in the size of individual droplets due to evaporation (condensation). The minimum and maximum radii do not remain constant either.

We will limit ourselves below to consideration of an initially homogeneous suspension, i.e., we assume that the initial unperturbed state is homogeneous over space and is characterized by some initial particle distribution over size $N_0(a_0)$ (the subscript 0 will denote unperturbed values of functions and parameters below). We also assume that the number of particles with radii from a to a + da per unit mixture volume is high and that motion of this set (fraction) of particles can be described by methods of the mechanics of heterogeneous media [8] as motion of a monodispersed continuum with characteristic particle radius a. We will then term all the monodispersed fractions in the mixture the polydispersed phase.* We will assume these fractions to be known beforehand and distinguish them by their unperturbed particle radius a_0. Thus a_0 will serve as the "number" or "index" of a fraction, thereby acting as some Lagrangian variable $\hat{a} = a_0$. Then the current radius $a = a(\hat{a}, \mathbf{r}, t)$.

We introduce $N(\hat{a}, \mathbf{r}, t)$, treating $N(\hat{a}, \mathbf{r}, t)d\hat{a}$ as the number of particles at the space-time "point" $(\mathbf{r}, t)$, initially having $a_0 = \hat{a}$: $a(\hat{a}, \mathbf{r}, t) = \hat{a} + a'(\hat{a}, \mathbf{r}, t)$; $N(\hat{a}, \mathbf{r}, t) = N_0(\hat{a}) + N'(\hat{a}, \mathbf{r}, t)$. Here and below the prime denotes small perturbations ($|a'| \ll a_0$, $|N'| \ll N_0$), the subscript 1 denotes parameters of the carrier phase, while 2 denotes parameters of the particle "phase."

The mass, velocity, and temperature of particles of "phase" 2 are functions of the Lagrangian variable $\hat{a}$:

$$m_2 = m_2(\hat{a}, \mathbf{r}, t) = \frac{4}{3}\pi a^3(\hat{a}, \mathbf{r}, t)\rho_2^0, \quad v_2 = v_2(\hat{a}, \mathbf{r}, t), \quad T_2 = T_2(\hat{a}, \mathbf{r}, t)$$

(ρ_2^0 = const is the true density of the particle material). It should be understood that the velocity and temperature of phase 1 do not depend on a. As for the intensities of interphase exchange between particles and gas, these depend on the internal variable $\hat{a}$, in view of the dependence of the latter on particle parameters.

The concepts of volume content α_j and mean density ρ_j of the j-th phase (j = 1, 2; $\alpha_1 + \alpha_2 = 1$) [8] can be generalized naturally to the case of a polydispersed suspension:

$$\alpha_2 = \int\limits_{\hat{a}_{\min}}^{\hat{a}_{\max}} \frac{4}{3}\pi a^3(\hat{a}, \mathbf{r}, t)N(\hat{a}, \mathbf{r}, t)\,d\hat{a}, \quad \rho_2 = \alpha_2\rho_2^0$$

($\hat{a}_{\min}$ and $\hat{a}_{\max}$ are the minimum and maximum particle radii in the unperturbed state).

2. Fundamental Equations. We will limit our further examination to one-dimensional motions with the condition of smallness of the medium parameter perturbations. We will illustrate the derivation of linear differential equations for the motion of the polydispersed mixture from the linear equations for a monodispersed mixture using the example of the equation of conservation of mass of the dispersed phase.

Let $d\rho_2 = m_2 N d\hat{a}$ be the mean density of the monodispersed fraction of particles charcterized by the parameter $\hat{a}$. The linearized equation of conservation of mass of this quasimonodispersed fraction can be written in the form [3] $\partial(\partial\rho_2')/\partial t + (d\rho_{20})\,\partial v_2'/\partial x = jN_0 d\hat{a}$. Integrating this equation over a from $\hat{a}_{\min}$ to $\hat{a}_{\max}$, we obtain

$$\frac{\partial\rho_2'}{\partial t} + \int\limits_{\hat{a}_{\min}}^{\hat{a}_{\max}} N_0(\hat{a})\,m_{20}(\hat{a})\,\frac{\partial v_2'}{\partial x}(\hat{a}, x, t)\,d\hat{a} = \int\limits_{\hat{a}_{\min}}^{\hat{a}_{\max}} N_0(\hat{a})\,j(\hat{a}, x, t)\,d\hat{a},$$

where ρ_2' is the perturbation of the mean density of the dispersed phase as a whole: $\rho_2' = \int\limits_{\hat{a}_{\min}}^{\hat{a}_{\max}} [m_2'(\hat{a}, x, t)N_0(\hat{a}) + m_{20}(\hat{a})N'(\hat{a}, x, t)]\,d\hat{a}$.

With similar considerations we can write the linearized system of equations for planar one-dimensional motion of a polydispersed mixture of vapor with droplets, generalizing the system of equations of motion of a monodispersed mixture of [3]:

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*The polydispersed "phase" is not a phase in the usual sense of this word, since we do not apply to it the concepts of mean phase velocity and temperature.

$$\frac{\partial\rho_1'}{\partial t}+\rho_{10}\frac{\partial v_1'}{\partial x}=-\int\limits_{\widehat{a}_{\min}}^{\widehat{a}_{\max}}\widetilde{N}_0\widetilde{j}d\widehat{a},\quad \frac{\partial\rho_2'}{\partial t}+\int\limits_{\widehat{a}_{\min}}^{\widehat{a}_{\max}}\widetilde{N}_0\widetilde{m}_{20}\frac{\partial\widetilde{v}_2'}{\partial x}\,d\widehat{a}=\int\limits_{\widehat{a}_{\min}}^{\widehat{a}_{\max}}\widetilde{N}_0\widetilde{j} \tag{2.1}$$

$$\rho_{10}\frac{\partial v_1'}{\partial t}+\frac{\partial p'}{\partial x}=-\int\limits_{\widehat{a}_{\min}}^{\widehat{a}_{\max}}\widetilde{N}_0\widetilde{f}d\widehat{a},\quad \widetilde{m}_{20}\frac{\partial\widetilde{v}_2'}{\partial t}=\widetilde{f}; \tag{2.2}$$

$$\rho_{10}c_1\frac{\partial T_1'}{\partial t}-\alpha_{10}\frac{\partial p'}{\partial t}=-\int\limits_{\widehat{a}_{\min}}^{\widehat{a}_{\max}}\widetilde{N}_0\widetilde{q}_{1\sigma}d\widehat{a},\quad \widetilde{m}_{20}c_2\frac{\partial\widetilde{T}_2'}{\partial t}=-\widetilde{q}_{2\sigma}; \tag{2.3}$$

$$\widetilde{q}_{1\sigma}+\widetilde{q}_{2\sigma}=-\widetilde{j}l_0,\quad p'=\frac{C_1^2}{\alpha_{10}\gamma_1}(\rho_1'+r\rho_2')+\frac{p_0}{T_0}T_1'. \tag{2.4}$$

Here Eq. (2.1) is the equation of conservation of mass of the carrier and dispersed phases; Eq. (2.2) is the equation of conservation of momentum of the gas and the dispersed particle fraction denoted by the parameter $\hat{a}$; Eq. (2.3) is the equation of heat influx for the gas and particles of the selected fraction; Eq. (2.4) is the equation of thermal balance on the surface of the particle identified by the parameter $\hat{a}$ and the equation of state of the gas; p is the pressure in the gas; $q_{j\sigma}$ is the thermal flux from phase j to the surface of an individual particle; ℓ is the specific heat of vapor formation; γ_1 is the adiabatic index of the gas; $r = \rho_{10}^0/\rho_2^0$ is ratio of the true phase densities; cj is the specific heat of the material of phase j (at constant pressure for the gas); C_1 is the speed of sound in the pure gas; quantities dependent on the parameter $\hat{a}$ are denoted by the symbol ~.

The system of Eqs. (2.1)-(2.4) is complete, if expressions are specified for the local mass, momentum, and heat fluxes: $\tilde{j}$, $\tilde{f}$ and $\tilde{q}_{j\sigma}$, respectively. According to [3], we have

$$\begin{gathered}\widetilde{f}=\widetilde{m}_{20}\left(\frac{v_1'-\widetilde{v}_2'}{\widetilde{\tau}_v^*}-\frac{v_1'}{\tau_A^*}\right),\quad \widetilde{j}=\frac{\widetilde{m}_{20}rc_1}{l_0}\frac{T_S'-\widetilde{T}_\sigma'}{\widetilde{\tau}_\sigma},\quad T_S'=\frac{T_0(1-r)}{l_0\rho_{10}^0}p',\\ \widetilde{q}_{1\sigma}=\widetilde{m}_{20}\frac{\rho_{10}c_1}{\rho_{20}}\frac{T_1'-\widetilde{T}_\sigma'}{\widetilde{\tau}_{T1}^*},\quad \widetilde{q}_{2\sigma}=\widetilde{m}_{20}c_2\frac{\widetilde{T}_2'-\widetilde{T}_\sigma'}{\widetilde{\tau}_{T2}^*},\end{gathered} \tag{2.5}$$

where T_S is the saturation temperature related to pressure by the Clapeyron—Clausius equation; T_σ is the surface temperature of an individual particle; τ are characteristic complex "times," which consider the nonsteady-state nature of interphase interaction processes and heat fluxes, of harmonic oscillations, at frequency ω, are defined by the expressions [9]

$$\begin{gathered}\widetilde{\tau}_v^*=\widetilde{\tau}_v\left[1+\frac{1-i}{\sqrt{2}}(\omega\widetilde{\tau}_{\mu1})^{1/2}-\frac{1}{9}i\omega\widetilde{\tau}_{\mu1}\right]^{-1},\quad \tau_A^*=-\frac{i}{r\omega},\\ \widetilde{\tau}_\sigma=\frac{1-r}{3(\gamma_1-1)}\sqrt{\frac{2\pi}{\gamma_1}\frac{\widehat{a}C_1^3}{\beta l_0^2}},\quad \widetilde{\tau}_{T1}^*=\frac{1}{3}\frac{\alpha_{10}}{\alpha_{20}}\widetilde{\tau}_{\lambda1}\eta_1(\widetilde{z}_1),\quad \widetilde{\tau}_{T2}^*=\frac{1}{15}\widetilde{\tau}_{\lambda2}\eta_2(\widetilde{z}_2),\\ \widetilde{\tau}_v=\frac{2}{9}\frac{\rho_2^0\widehat{a}^2}{\mu_1},\quad \widetilde{\tau}_{\mu1}=\frac{\rho_1^0\widehat{a}^2}{\mu_1},\quad \widetilde{\tau}_{\lambda j}=\frac{\widehat{a}^2}{\varkappa_j},\quad \widetilde{z}_j=\frac{1-i}{\sqrt{2}}(\omega\widetilde{\tau}_{\lambda j})^{1/2}\quad (j=1,2),\\ \eta_1(z)=\frac{1}{1+z},\quad \eta_2(z)=\frac{5[3z-(3+z^2)\operatorname{th}z]}{z^2(\operatorname{th}z-z)},\quad \varkappa_j=\frac{\lambda_j}{\rho_j^0c_j}.\end{gathered} \tag{2.6}$$

Here μ_1 is the dynamic viscosity of the gas; λ is the thermal conductivity coefficient; β is the accommodation coefficient. The closed system of Eqs. (2.1)-(2.6), obtained with the assumption of smallness of the volume content of the suspended phase $\alpha_2 << 1$, can be used to analyze the acoustical properties of polydispersed mixtures of vapor with droplets of gas with particles.

3. Dispersion Relationships. We will seek solutions of system (2.1)-(2.6) in the form of progressive waves for perturbations of the parameters χ':

$$\chi' = A_\chi \exp\left[i\left(k_* x - \omega t\right)\right] = A_\chi \exp\left(-k_{**} x\right) \exp\left[i\left(kx - \omega t\right)\right]$$
$$\left(k_* = k + ik_{**},\quad C_p = \omega/k,\ C_g = d\omega/dk,\quad \sigma = 2\pi C_p k_{**}/\omega\right), \tag{3.1}$$

where A_χ is the complex amplitude of the perturbation in the parameter χ; i is the square root of minus one; k_* is the complex wave number; k_{**} is the linear attenuation coefficent; Cp, Cg, and σ are the phase velocity, group velocity, and wavelength attenuation constant, respectively.

Here, in contrast to the monodispersed case [3], the perturbations v'_2, T'_2, and T'_σ depend not only on (x, t), but also on the internal dispersed phase parameter a. Thus, the amplitudes of the perturbations A_{V2}, A_{T2}, $A_{T\sigma}$ are functions of $\hat{a}$, which in accord with the notation introduced above will be denoted below by the symbol ~.

We denote by $\langle\tilde{f}\rangle$ the action on $\tilde{f}$ of the linear operator:

$$\langle\tilde{f}\rangle = \left(\int\limits_{\widehat{a}_{\min}}^{\widehat{a}_{\max}} \tilde{N}_0 \tilde{f} \widehat{a}^3\, d\widehat{a}\right) \Bigg/ \left(\int\limits_{\widehat{a}_{\min}}^{\widehat{a}_{\max}} \tilde{N}_0 \widehat{a}^3 d\widehat{a}\right) = \frac{1}{\rho_{20}} \int\limits_{\widehat{a}_{\min}}^{\widehat{a}_{\max}} \tilde{N}_0 \tilde{m}_{20} \tilde{f} d\widehat{a}.$$

In view of the definitions used, if f is not dependent on $\hat{a}$, then $\langle f\rangle \equiv f$. Substituting Eq. (3.1) in Eqs. (2.1)-(2.6), we have

$$-i\omega A_{\rho 1} + ik_* \rho_{10} A_{v1} + \frac{rc_1}{l_0} \rho_{20} \left\langle \frac{A_{TS} - \tilde{A}_{T\sigma}}{\tilde{\tau}_\sigma} \right\rangle = 0, \tag{3.2}$$
$$-i\omega A_{\rho 2} + ik_* \rho_{20} \langle \tilde{A}_{v2} \rangle - \frac{rc_1}{l_0} \rho_{20} \left\langle \frac{A_{TS} - \tilde{A}_{T\sigma}}{\tilde{\tau}_\sigma} \right\rangle = 0,$$
$$-i\omega \rho_{10} A_{v1} + ik_* A_p + \rho_{20} \left[\left\langle \frac{A_{v1} - \tilde{A}_{v2}}{\tilde{\tau}_v^*} \right\rangle - \frac{A_{v1}}{\tau_A^*} \right] = 0,$$
$$-i\omega \rho_{10} c_1 A_{T1} + i\omega \alpha_{10} A_p + \rho_{10} c_1 \left\langle \frac{A_{T1} - \tilde{A}_{T\sigma}}{\tilde{\tau}_{T1}^*} \right\rangle = 0;$$

$$-i\omega \tilde{A}_{v2} - \frac{A_{v1} - \tilde{A}_{v2}}{\tilde{\tau}_v^*} + \frac{A_{v1}}{\tau_A^*} = 0,\quad -i\omega \tilde{A}_{T2} + \frac{\tilde{A}_{T2} - \tilde{A}_{T\sigma}}{\tilde{\tau}_{T2}^*} = 0, \tag{3.3}$$
$$\frac{A_{T1} - \tilde{A}_{T\sigma}}{\tilde{\tau}_{\sigma 1}^*} + \frac{\tilde{A}_{T2} - \tilde{A}_{T\sigma}}{\tilde{\tau}_{\sigma 2}^*} - \frac{\tilde{A}_{T\sigma} - A_{TS}}{\tilde{\tau}_\sigma} = 0;$$

$$A_p - \frac{C_1^2}{\alpha_{10} \gamma_1} \left(A_{\rho 1} + rA_{\rho 2}\right) - \frac{p_0}{T_0} A_{T1} = 0,\quad A_{TS} - \frac{T_0 (1-r)}{\rho_{10}^0 l_0} A_p = 0 \tag{3.4}$$
$$\left(\tilde{\tau}_{\sigma 1}^* = \frac{\alpha_{20}}{\alpha_{10}} \tilde{\tau}_{T1}^*,\quad \tilde{\tau}_{\sigma 2}^* = \frac{\rho_{10}^0 c_1}{\rho_2^0 c_2} \tilde{\tau}_{T2}^* \right).$$

It follows from Eq. (3.3) that the amplitudes $\tilde{A}_{V2}$, $\tilde{A}_{T2}$, and $\tilde{A}_{T\sigma}$ can be expressed in terms of characteristic relaxation times which are known functions of $\hat{a}$ (see Eq. (2.6)), and the amplitudes A_{V1}, A_{T1}, A_{TS} which do not depend on $\hat{a}$:

$$\tilde{A}_{v2} = \frac{1 - \tilde{\tau}_v^* / \tau_A^*}{1 - i\omega \tilde{\tau}_v^*} A_{v1},\quad \tilde{A}_{T2} = \frac{1}{1 - i\omega \tilde{\tau}_{T2}^*} \tilde{A}_{T\sigma}, \tag{3.5}$$
$$\tilde{A}_{T\sigma} = \left[\frac{1}{\tilde{\tau}_\sigma} + \frac{1}{\tilde{\tau}_{\sigma 1}^*} + \frac{1}{\tilde{\tau}_{\sigma 2}^*} - \frac{1}{\tilde{\tau}_{\sigma 2}^* \left(1 - i\omega \tilde{\tau}_{T2}^*\right)} \right]^{-1} \left(\frac{1}{\tilde{\tau}_{\sigma 1}^*} A_{T1} + \frac{1}{\tilde{\tau}_\sigma} A_{TS} \right).$$

Substituting Eq. (3.5) in the corresponding expressions of Eq. (3.2), all the perturbation amplitudes can be brought out from within the averaging operator ⟨⟩. As a result, from Eqs. (3.2), (3.4), we obtain a homogeneous system of six linear algebraic equations in the amplitudes $A_{\rho 1}$, $A_{\rho 2}$, A_{V1}, A_p, A_{T1}, A_{TS}.

The dispersion dependence of wave number on oscillation frequency ω with which we are concerned can be found from the condition of a nontrivial solution of this homogeneous system and can be written in the form [3]

$$\left(C_1 k_* / \omega\right)^2 = V(\omega)\, \Theta(\omega), \tag{3.6}$$

where $V(\omega)$ and $\Theta(\omega)$ are complex functions of frequency ω, the first of which is responsible for description of dispersion and dissipation due to interphase friction, while the second is related to interphase heat exchange. They depend on the thermophysical properties of the phases and the spectral composition of the mixture:

$$V(\omega)=1+m\frac{(\alpha_{10}-r)\langle\widetilde{g}\rangle-\alpha_{10}r}{1+mr\langle\widetilde{g}\rangle},\quad \widetilde{g}=\frac{1-\widetilde{\tau}_v^*/\tau_A^*}{1-i\omega\widetilde{\tau}_v^*};\tag{3.7}$$

$$\Theta(\omega)=1-mr\frac{(\gamma_1-1)\langle\widetilde{e}_3\rangle-\frac{1-r}{\hat{l}}\left[2\langle\widetilde{e}_1\rangle-\frac{1-r}{(\gamma_1-1)\hat{l}}\langle\widetilde{e}_2\rangle\right]+\frac{mr}{\gamma_1-1}\left(\frac{1-r}{\hat{l}}\right)^2[\langle\widetilde{e}_1\rangle^2-\langle\widetilde{e}_2\rangle\langle\widetilde{e}_3\rangle]}{1-mr\langle\widetilde{e}_3\rangle},\tag{3.8}$$

$$\widetilde{e}_1=\left[i\omega\left(\widetilde{\tau}_\sigma+\widetilde{\tau}_{\sigma1}^*\right)+\frac{c_2}{rc_1}\frac{\omega^2\widetilde{\tau}_\sigma\widetilde{\tau}_{\sigma1}^*}{1-i\omega\widetilde{\tau}_{T2}^*}\right]^{-},$$

$$\widetilde{e}_2=\frac{1}{i\omega\widetilde{\tau}_\sigma}\left(1-i\omega\widetilde{\tau}_{\sigma1}^*\widetilde{e}_1\right),\quad \widetilde{e}_3=\frac{1}{i\omega\widetilde{\tau}_{\sigma1}^*}\left(1-i\omega\,\widetilde{\tau}_\sigma\widetilde{e}_1\right)$$

($m = \rho_{20}/\rho_{10}$ is the relative mass content of particles, $\hat{\ell} = \ell/C_1^2$ is the dimensionless heat of vapor formation).

Dispersion equations (3.6)-(3.8) describe the acoustical properites of polydispersed vapor and gas suspensions with an arbitrary initial distribution of particles over size. In the special case of a monodispersed suspension, where the probability density is a Dirac δ-function, i.e., $N_0(\hat{a}) = n_0\delta(\hat{a} - a_0)$, we have $\langle\widetilde{g}\rangle = g(a_0, \omega)$, $\langle\widetilde{e}_j\rangle = e_j(a_0, \omega)$ $(j = 1, 2, 3)$ and Eqs. (3.6)-(3.8) naturally transform to the expressions known in [3]. For $m = 0$, when there are no particles, $V = \Theta = 1$, i.e., wave dispersion and dissipation are absent from a gas without particles. For a polydispersed mixture of gas with solid particles (no phase transitions, $\tau_\sigma = \infty$) the function $\Theta(\omega)$ (Eq. (3.8)) in dispersion relationship (3.6) has a significantly simpler form:

$$\Theta(\omega)=1+(\gamma_1-1)\frac{H_T(\omega)}{1+H_T(\omega)},\quad H_T=m\frac{c_2}{c_1}\langle\widetilde{h}_T\rangle,\tag{3.9}$$

$$\widetilde{h}_T=\left(1-i\omega\widetilde{\tau}_T^*\right)^{-1},\quad \widetilde{\tau}_T^*=\frac{c_2}{rc_1}\left(\widetilde{\tau}_{\sigma1}^*+\widetilde{\tau}_{\sigma2}^*\right)\approx\frac{c_2}{rc_1}\widetilde{\tau}_{\sigma1}^*,$$

which also agrees with the known result for monodispersed gas suspensions [3].

At moderate gas phase pressures, $r \ll 1$. In this case, also considering $\alpha_2 \ll 1$, we can obtain from Eq. (3.7) a simpler expression for the dispersive-dissipative function $V(\omega)$:

$$V(\omega)=1+H_v(\omega),\quad H_v=m\langle\widetilde{h}_v\rangle,\tag{3.10}$$

$$\widetilde{h}_v=\left(1-i\omega\widetilde{\tau}_v^{*0}\right)^{-1},\quad \widetilde{\tau}_v^{*0}=\widetilde{\tau}_v\left[1+\frac{1-i}{\sqrt{2}}(\omega\widetilde{\tau}_{\mu1})^{1/2}\right].$$

If there are no phase transitions, then it is reasonable to consider propagation of acoustic waves in a gas suspension consisting of a finite number of particle fractions, distinguished not only by their size, but also their thermophysical properties. Examples of such media are various types of smoke, gas suspensions contaminated by impurities, etc. In this case the relaxation "times" for each dispersed phase will differ not only because of particle size, but also because of thermophysical properties of the phases. The dispersion relationship describing sound propagation in a multiphase gas suspension was presented in [7]. It can be derived from a system of the type of Eqs. (2.1)-(2.6) and has the form of Eqs. (3.6), (3.9), (3.10), where by H_v and H_T we understand

$$H_v=\sum_{k=2}^{N}m_kh_{vk},\quad H_T=\sum_{k=2}^{N}m_k\frac{c_k}{c_1}h_{Tk},\quad \sum_{k=2}^{N}m_k=m.\tag{3.11}$$

Here c_k, m_k are the specific heat of the particle material and the relative mass content of the k-th dispersed phase $(k = 2, \ldots, N)$; h_{vk}, and h_{Tk} are obtained when in the corresponding expressions for $\widetilde{h}_v$, $\widetilde{h}_T$ and $\widetilde{\tau}$ in Eqs. (3.9), (3.10), (2.6), we take $\hat{a} = a_k$, $\rho_2^0 = \rho_k^0$, $c_2 = c_k$ (ρ_k^0 is the density of the particle material in dispersed phase k).

It is clear from Eqs. (3.6), (3.9)-(3.11) how the dispersion relationship can be constructed when the gas suspension consists of a finite number of particle types differing in thermophysical properties, and in each type there exists a distribution of particles over size with a function $N_{k0}(\hat{a}_k)$ (where $\hat{a}_k$ is the Lagrangian parameter of polydispersed phase k). The functions H_V and H_T appearing in Eqs. (3.9), (3.10) are as follows:

$$H_v = \sum_{k=2}^{N} m_k \langle \widetilde{h}_{vk} \rangle_k, \quad H_T = \sum_{k=2}^{N} m_k \frac{c_k}{c_1} \langle \widetilde{h}_{Tk} \rangle_k \tag{3.12}$$

($\langle\rangle_k$ is the averaging operator with function $N_{k0}(\hat{a}_k)$).

Of special interest is the aerosol-type suspension widely found in practice with a low particle content (m << 1). Preserving only terms linear in m in Eqs. (3.6)-(3.8) and considering that r << 1, we find the simplest dispersion relationship for one-component aerosols with phase transitions:

$$\begin{gathered} C_1 k_*/\omega = 1 + \frac{m}{2}\left(V^0(\omega) + \Theta^0(\omega)\right), \quad V^0 = \langle \widetilde{h}_v \rangle, \\ \Theta^0 = r\left[\frac{1}{\bar{l}}\left(2\langle \widetilde{e}_1 \rangle - \frac{1}{(\gamma_1 - 1)\bar{l}}\langle \widetilde{e}_2 \rangle\right) - (\gamma_1 - 1)\langle \widetilde{e}_3 \rangle\right] \end{gathered} \tag{3.13}$$

($\tilde{e}_j$ and $\tilde{h}_V$ are given in Eqs. (3.8), (3.10)). Here, in contrast to Eqs. (3.6)-(3.8), we have additiveness of the contributions of interphase friction and heat-mass exchange to dispersion and dissipation of perturbations. However, in view of the linearity of the operator $\langle\rangle$, it is evident that Eq. (3.13) can be obtained from the corresponding relationship for a monodispersed aerosol by simple integration over the masses of the fractions.

If in Eq. (3.13) we replace Θ^0 by $(\gamma_1 - 1)H_T/m$, then we find a dispersion relationship for describing the acoustical properties of polydispersed aerosols of the gas—solid particle type. We note that it is applicable for any oscillation frequencies satisfying the requirement of acoustical homogeneity of the suspension. With the aid of this relationship one can easily obtain explicit expressions for velocity and attenuation coefficient of weak monochromatic waves in an aerosol mixture of gas and solid particles. Testing shows that at sufficiently low frequencies, where the role of nonsteady-state interphase interaction effects is small, these expressions coincide with those obtained for special cases using other considerations in [6].

4. Analysis of Results. To study the features of sound propagation in polydispersed suspensions, it is sufficient to consider the simplest case of a mixture without phase transitions, the acoustical properties of which are described by dispersion relationship (3.6), (3.9), (3.10). To supplement those introduced above, we now use the following dimensionless quantities and parameters characterizing the composition, thermophysical and acoustic properties of such a mixture:

$$\begin{gathered} \mathrm{Pr}_1 = \frac{\mu_1 c_1}{\lambda_1}, \quad \bar{c} = \frac{c_2}{c_1}, \quad \tau_{v*} = \frac{2}{9}\frac{\rho_2^0 a_*^2}{\mu_1}, \quad \Omega = \omega\tau_{v*}, \quad \eta = \lg\Omega, \\ \overline{N}(\bar{a}) = \frac{a_*}{n_0} N_0(\hat{a}) \quad \left(\int_{\bar{a}_{\min}}^{\bar{a}_{\max}} \overline{N}(\bar{a})\, d\bar{a} = 1\right), \quad \bar{a} = \frac{\hat{a}}{a_*}, \quad K = k_* C_1 \tau_{v*}, \\ K = K_1 + iK_2, \quad \overline{C}_p = C_p/C_1, \quad \overline{C}_g = C_g/C_1, \ \sigma = 2\pi K_2/K_1. \end{gathered} \tag{4.1}$$

Here a_* is a representative radius, selection of which will be discussed below; $\overline{N}(\bar{a})$ is the probability density in the initial state.

The dimensionless relationships establish the equivalence of all gas suspensions with identical thermophysical properties, identical dimensionless spectrum, and identical mass content m. For example, for equality of all other dimensionless parameters, all monodispersed suspensions are equivalent, independent of particle radius; also equivalent are all suspensions with a uniform spectrum having identical ratios $\bar{a}_{\max}/\bar{a}_{\min}$, etc. At the same time, if two suspensions differ in their dimensionless spectra with other parameters equal, then generally speaking, their dependences of wave number on dimensionless frequency will differ functionally.

Despite the obviousness of the above and the fact that it has been confirmed experimentally [6], in the literature one meets with the opinion ([10], for example) that by introducing some effective radius one can achieve a situation in which the acoustical properties of the polydispersed medium will be describable within the framework of a monodispersed model.

We will consider high- and low-frequency asymptotes of the dimensionless wave number following from Eqs. (3.6), (3.9), (3.10), (4.1) as $\Omega \to \infty$ and $\Omega \to 0$. Terms ~r are negligible in comparison to unity; we will consider $\bar{c}$, Pr_1, m finite. The terms of the expansion of concern to us have the form

$$(K/\Omega)_{\substack{\Omega\to\infty \\ (r\ll 1)}} \sim 1 + \frac{m}{2}\left\{\frac{3}{2}\sqrt{r}\,(1+i)\,\psi_\infty\left(\sqrt{\Pr_1}\right)\left(\bar{a}_{3,2}^2\Omega\right)^{-1/2} + \right. \tag{4.2}$$

$$\left. + i\psi_\infty(\Pr_1)\left(\bar{a}_{3,1}^2\Omega\right)^{-1}\right\}, \quad \psi_\infty(y) = 1 + 2(\gamma_1 - 1)(3y)^{-1};$$

$$(K/\Omega)_{\substack{\Omega\to 0 \\ (r\ll 1)}} \sim \bar{C}_e^1\left\{1 + \frac{m}{2(1+m)}\,i\psi_0(\Pr_1)\left(\bar{a}_{5,3}^2\Omega\right)\right\}, \tag{4.3}$$

$$\psi_0(y) = 1 + \frac{3}{2}(\gamma_1 - 1)\left[\frac{\bar{c}\bar{C}_e(1+m)}{1+m\bar{c}}\right]^2 y, \quad \bar{C}_e = \left[\frac{1+m\bar{c}}{(1+m)(1+m\bar{c}\gamma_1)}\right]^{1/2}.$$

Here $\bar{C}_e$ is the equilibrium dimensionless speed of sound in the mixture; $a_{i,j}$ are mean radii defined by

$$a_{i,j} = \left\{\left[\int\limits_{\hat{a}_{\min}}^{\hat{a}_{\max}} N_0(\hat{a})\,\hat{a}^i d\hat{a}\right] \Big/ \left[\int\limits_{\hat{a}_{\min}}^{\hat{a}_{\max}} N_0(\hat{a})\,\hat{a}^j d\hat{a}\right]\right\}^{\frac{1}{i-j}}, \quad i \neq j, \quad \bar{a}_{i,j} = \frac{a_{i,j}}{a_*}. \tag{4.4}$$

The mean radii introduced in this manner lie on the segment $[\hat{a}_{\min}, \hat{a}_{\max}]$ and have the symmetry property $a_{i,j} = a_{j,i}$.

According to Eqs. (4.2), (4.3) (Eq. (4.2) can be used at frequencies which do not disturb the acoustical homogeneity of the medium [3]), sound attenuation in the polydispersed mixture at high or low frequencies is not determined directly by the form of the function $N_0(\hat{a})$, but rather by the integral characteristics $a_{i,j}$. Therfore, if we choose as the dedimensionalization parameter a_* the radius $a_{3,2}$ (or, if r is very small, $a_{3,1}$), then in light of that choice $\bar{a}_{3,2} = 1$ ($\bar{a}_{3,1} = 1$) and attenuation of high frequency perturbations in the polydispersed mixture with arbitrary spectrum will coincide with attenuation in a monodispersed mixture with $a_0 = a_*$. The same can be said of the role of $a_{5,3}$ for low frequencies.

We will consider the effect of choice of the dedimensionalization parameter a_* on dimensionless functions of the wave number. As an example we will consider the graph of the wavelength attenuation decrement $\sigma(\log \Omega) = \sigma(\eta)$, which for a monodispersed suspension has a characteristic bell-shaped form (Fig. 1, dashed line). Let the dimensionless function $\bar{N}(\bar{a})$ be fixed. Then the graphs of $\sigma(\eta_1)$ and $\sigma(\eta_2)$ ($\eta_j = \log \Omega_j$, Ω_j being the dimensionless frequency in the sense of the dedimensionalization parameter a_{*j} (j = 1, 2)) will obviously transform into each other upon simple displacement along the abscissa η, since $\eta_1 = \lg\Omega_1 = \lg\Omega_2 + 2\lg(a_{*1}/a_{*2}) = \eta_2 + 2\lg(a_{*1}/a_{*2})$.

We now introduce the concept of a reference curve $\sigma_r(\eta)$. It follows from the definition of Eq. (4.4) that for a monodispersed mixture with radius a_0, all $a_{i,j} = a_0$. We term the curve of the function $\sigma(\eta)$ for the monodispersed medium the reference $\sigma = \sigma_r(\eta)$, if for the dedimensionalization parameter a_*, the radius a_0 is used. Hence for a monodispersed mixture for any $i \neq j$ we have $\sigma(\eta_{i,j}) = \sigma_r(\eta)$ ($\eta_{i,j} = \lg\Omega_{i,j}$, $\Omega_{i,j}$ being the dimensionless frequency in the sense of the dedimensionalization parameter $a_{i,j}$).

We will consider the graph of $\sigma(\eta)$ for a polydispersed suspension as compared to the reference. For a_* let us choose the radius $a_{3,2}$. Then in the high-frequency region, the graph of $\sigma(\eta_{3,2})$ and $\sigma_r(\eta_{3,2})$ coincide asymptotically. At low frequencies, in view of the Helder inequality $\bar{a}_{5,3} = a_{5,3}/a_{3,2} \geqslant 1$ and the monotonic growth of $\sigma(\eta)$ we have $\sigma(\eta_{3,2}) \geqslant \sigma_r(\eta_{3,2})$. If we choose $a_* = a_{5,3}$ then although asymptotically coinciding at low frequencies the graphs of $\sigma(\eta_{5,3})$ and $\sigma_r(\eta_{5,3}) = \sigma_r(\eta_{3,2})$ differ at high frequencies, while from $\bar{a}_{3,2} = a_{3,2}/a_{5,3} \leqslant 1$ and the monotonic decrease of $\sigma(\eta)$ we obtain $\sigma(\eta_{5,3}) \geqslant \sigma_r(\eta_{5,3})$. If we choose a_* from the condition

$a_{3,2} \leqslant a_* \leqslant a_{5,3}$, $a_* = a_0$, then in both high and low frequency regions $\sigma(\eta) \geqslant \sigma_r(\eta)$. With such a choice of a_* there is a correspondence in the characteristic regions of change in the parameter η where the functions $\sigma(\eta)$ and $\sigma_r(\eta)$ reach their extremal values.

Figures 1-6 show results of calculating sound speed and attenuation in gas suspensions with parameters $m = 1$, $\gamma_1 = 1.4$, $\mathrm{Pr}_1 = 0.72$, $r = 4.3 \cdot 10^{-4}$, $\bar{c} = 0.87$, corresponding to mixtures of aluminum dust with air at a pressure of 0.1 MPa and temperature of 293 K. For the dedimensionalization parameter we take the radius $a_{4,3}$, which according to the Helder inequality lies in the interval $[a_{3,2}, a_{5,3}]$. In this case, $\bar{N}(\bar{a})$ satisfies the normalization condition $a_{1,3} = 1$, i.e., we have a dual normalization $\int\limits_{\bar{a}_{\min}}^{\bar{a}_{\max}} \bar{N}(\bar{a})\bar{a}^4\,d\bar{a} = \int\limits_{\bar{a}_{\min}}^{\bar{a}_{\max}} \bar{N}(\bar{a})\bar{a}^3\,d\bar{a}$, $\int\limits_{\bar{a}_{\min}}^{\bar{a}_{\max}} \bar{N}(\bar{a})\,d\bar{a} = 1$.

As an illustrations we choose the following functions defined on the interval $[\bar{a}_{\min}, \bar{a}_{\max}]$:

$$1)\ \bar{N}(\bar{a}) = \text{const},\quad 2)\ \bar{N}(\bar{a}) = \text{const} \cdot \bar{a}^{-3},$$

$$3)\ \bar{N}(\bar{a}) = \bar{n}_1\delta(\bar{a} - \bar{a}_1) + \bar{n}_2\delta(\bar{a} - \bar{a}_2),$$

which characterize two forms of uniform distribution (1, over radius; 2, over mass) and a two-fraction composition (distribution 3). In view of the dual normalization $\bar{N}(\bar{a})$ of distributions 1, 2 can be specified by one independent parameter $\psi = a_{\max}/a_{\min}$, while that of distribution 3 requires two parameters $\psi = a_2/a_1$ and, for example, $\varphi = m_1/m_2 = (n_1a_1^3)/(n_2a_2^3)$:

1) $\text{const} = \frac{4}{5}(\psi^5 - 1)/[(\psi^4 - 1)(\psi - 1)]$, $\bar{a}_{\min} = \frac{5}{4}(\psi^4 - 1)/(\psi^5 - 1)$, $\bar{a}_{\max} = \psi\bar{a}_{\min}$,

2) $\text{const} = 8\psi^2/[(\psi + 1)^3(\psi - 1)]$, $\bar{a}_{\min} = 2/(\psi + 1)$, $\bar{a}_{\max} = \psi\bar{a}_{\min}$,

3) $\bar{n}_2 = 1/(1 + \varphi\psi^3)$, $\bar{n}_1 = 1 - \bar{n}_2$, $\bar{a}_1 = (\varphi + 1)/(\varphi + \psi)$, $\bar{a}_2 = \psi\bar{a}_1$.

The dependences of phase velocity $\bar{C}_p$ and attenuation decrement σ on dimensionless frequency $\Omega_{4,3}$ for suspensions with distributions 1 and 2 are shown in Figs. 1, 2, respectively. The dashed lines are the limiting cases $\psi = 1$, corresponding to a monodispersed mixture (ref-

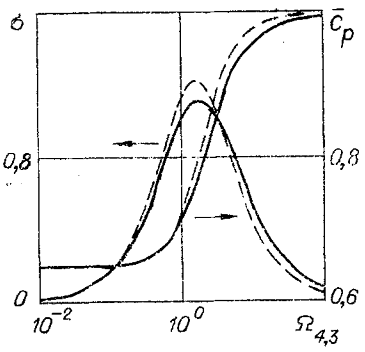

Fig. 1

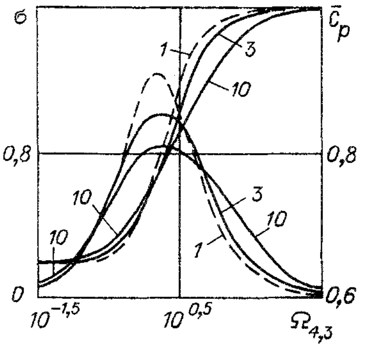

Fig. 2

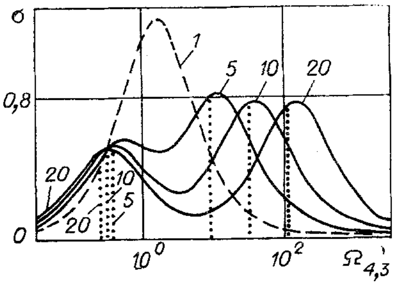

Fig 3

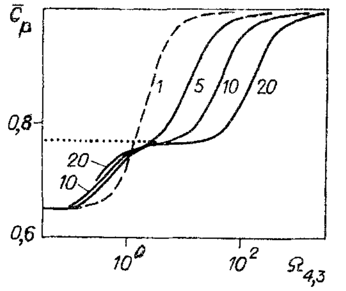

Fig. 4

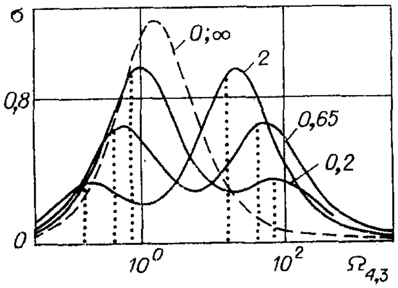

Fig. 5

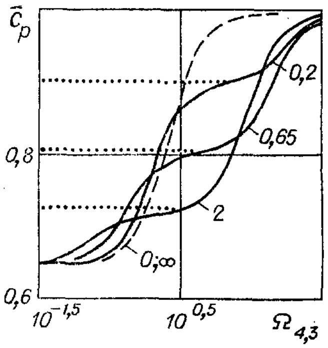

Fig. 6

erence curves). For distribution 1, aside from $\psi = 1$, only the case $\psi = \infty$ (solid lines) is shown, since even in those limiting cases $\psi = 1$ and $\psi = \infty$ do not differ greatly; the numbers along the lines of Fig. 2 are values of the parameter ψ. It is obvious that with increase in ψ (broadening of the spectrum) the maximum wavelength attenuation decrement decreases markedly, while the dependence of the speed of sound on frequency becomes less intense.

Figures 3-6 show dispersion and dissipation of weak waves in a gas suspension with discrete spectrum 3. Figures 3, 4 were constructed for fixed $\varphi = 1$ and variation of the parameter ψ (numbers along lines). For Figs. 5, 6, we have a fixed $\psi = 10$, while the numbers along the lines denote values. The vertical dashed lines of Figs. 3, 5, are characteristic frequencies $\Omega_2 = \bar{a}_2^{-2}$ and $\Omega_1 = \bar{a}_1^{-2} (\bar{a}_1 \leqslant \bar{a}_2)$, while the horizontal lines of Figs. 4, 6 are "shelf" levels, corresponding to the equilibrium speed of sound in the small particle space $\bar{C}_{e1} = \{(1 + m_1\bar{c})/[(1 + m_1)(1 + m_1\bar{c}\gamma_1)]\}^{-1/2}$, $m_1 = m\varphi/(1 + \varphi)$.

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